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A theoretical study on the relationship of radius of maximum wind, distribution of pressure and wind in a cyclonic storm

A Muthuchami[1] and P Dhanavanthan[2]*

[1]Regional Meteorological Centre, Chennai 600 006, Tamil Nadu, India
[2]University of Madras, Chennai - 600 005, Tamil Nadu, India

E-mail : pdhanvanthan@hotmail.com

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Abstract : In this paper, an attempt is made to explain the asymmetrical distribution of wind and pressure field in a cyclonic storm by assuming the spiral flow by incorporating the radial flow. Based on this hypothesis, it is found that the circular motion seems to occur only at radius of maximum wind(RMW) and hence, gradient wind balance is achieved only at RMW. The maximum vertical velocity does not occur at RMW. The pressure gradient is steep in the proximity of RMW, and the pressure distribution is symmetric around the centre only at RMW. Out side the RMW pressure and pressure gradient are not symmetrically distributed. The spiral is assumed to be governed by the relation $r = R_0 \exp[k/(\theta+\theta_0)]$ where R_0 is the RMW and K is a constant which determines the shape of the spiral. The maximum wind is not only function of pressure gradient at RMW but also of radius of Maximum wind.

Keywords : Cvclonic storm, radius of maximum wind, distribution of pressure and wind

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1. Introduction

The well recognized signature of a tropical cyclone has the characteristic of rapid decrease in surface pressure and increase in surface winds towards the centre with the heavy rain area and the storm lashed seas and relatively calm area. A mature tropical cyclone has a core region typically extending three to six times the radius of maximum winds from the cyclone centre. The core region is one of high inertial stability and rotational froude number flow and is dominated by convective process. Gray and Shea [1] have documented the main horizontal features of azimuthal and radial wind fields in the core region. They found that the resulting azimuthal and radial wind so composited have distinct wave number 1 asymmetry. A narrow

*Corresponding Author :
On leave from : Department of Physics, Hindu College, Sonipat 131-001, Haryana, India

belt of very high winds near radius of maximum wind surrounds a quasi – circular and calm eye region. The strong radial gradient of 700 hPa height roughly coincides with the maximum swirling wind which suggests that wind may be in gradient balance with the mass field. The gradual evolution and circularity of lower troposphere vortex supports the notion of gradient balance. Although the idea has some observational basis [3, 5, 16-17] the possibility of deviation from the balance where radial acceleration are large such as when the frictional inflow decelerates. Gupta and Muthuchami [2] composited the upper air wind observations and studied the vertical structure of cyclonic storm of May 1990. Under the eye wall, balanced models seem to explain many aspects of the evolution of axis symmetric tropical cyclones [10,13,15] Schubert and Hack (1982). Shapiro [12] concluded that the interaction between the asymmetries and the symmetric hurricane vortex at early times depend on realistic features of the model hurricane and not on interactions between the asymmetries and the boundary layer, which possibly depend on the convective parameterization. In particular, the changes in the symmetric wind tendency due to the asymmetry can be most simply explained by a combination of horizontal advection and damping of wave activity.

The heating induced inflow can supply an excess of angular momentum beyond that required to balance frictional loss [9]. One widely used model of lower troposphere wind profile in tropical cyclones is that it increases linearly with radius inside the (RMW) and decreases inversely as the square root of radius outside the RMW [1,4]. By assuming the pressure as exponential function of radius [7,11] derived the maximum wind as a function of pressure difference between environment and the cyclone center. Mishra and Gupta [6] compiled the storm statistics over north Indian Ocean and derived an expression for maximum wind from the pressure defect.

This paper explains the non uniformity in wind field and pressure field around the centre of the storm. In this case it is assumed that the air parcel approaches the centre along the spiral path which is the result of the introduction of radial wind in the system. On the assumption of structure of cyclone the relation between the radius of maximum wind with pressure gradient is determined and the results are presented. Section. 2 deals with data utilised followed by the analysis of the model in section. 3. The results and conclusion are given in section 4 and 5 respectively.

2. Data

After 1990 all the storms with clear eye structure which crossed near the observatories where the hourly or autographic charts of rainfall and wind observations (surface level) are available are taken for the study. The hourly observations are taken during storm period and sent to Area Cyclone Warning Centres (ACWC) on real time basis. These data are available in ACWC (Chennai). The autographic charts are available in the climatological section of the Regional Meteorological Centre, Chennai. The rainfall and wind data of five of cyclonic storms which crossed in southern coast of India is considered for the discussion

3. Theoretical Analysis

The main objective of this paper is to propose a simple mechanism that may yield an explanation of the main feature of cyclone like wind distribution, convergence area and pressure field and their relation with its wind field. This model is little different from the Rankine vortex which most of the authors used to choose.

If we assume that the isobars are circular, motion is not cross isobaric and steady, then the equation of motion is

$$\frac{dV_r}{dt} - fV_\theta - \frac{V_\theta^2}{r} = -\frac{\partial P}{\rho \partial r} \tag{1}$$

$$\frac{dV_\theta}{dt} - fV_r + V_\theta V_r = -\frac{\partial P}{\rho r \partial \theta} \tag{2}$$

where V_r, V_θ are radial and azimuthal velocities respectively and ρ is density of the atmosphere. Since motion is not cross isobaric, $\frac{dV_r}{dt} = 0$ and therefore (1) yields

$$-fV_\theta - \frac{V_\theta^2}{r} = \frac{\partial p}{\rho \partial r} \tag{3}$$

which is a gradient wind balance.

Since the radial velocity is zero and pressure is symmetric with respect to centre under the assumption, $\frac{dV_r}{dt} = 0$ gives

$$\frac{V_\theta \partial V_\theta}{r \partial \theta} + W\frac{\partial V_\theta}{\partial z} = 0 \tag{4}$$

where W is the vertical velocity and z is the height of the parcel of air. As motion is steady, that is

$$\frac{\partial V_\theta}{\partial t} = 0.$$

Further, $$\frac{\partial z}{\partial \theta} = \frac{-Wr}{V_\theta} \tag{5}$$

where $\frac{\partial z}{\partial \theta}$ is the rate of change of vertical displacement of the air parcel with respect to change in angular displacement. It suggests that whenever $W > 0$, $\frac{\partial z}{\partial \theta} < 0$ and hence in vertical upward motion, the parcel moves downward and in the similar way when $W > 0$, the parcel moves upward which is a contradiction.

In a circular isobaric field, steady gradient flow is not possible in an unstable atmosphere, that is, in a circular isobaric field, gradient flow is possible only when the vertical motion is absent. It is assumed that the motion is steady. It may be considered that the air mass is injected into the circulation at the outer region towards the centre of the cyclone. Figure (1) gives schematic representation of the idealized model showing horizontal cross sections.

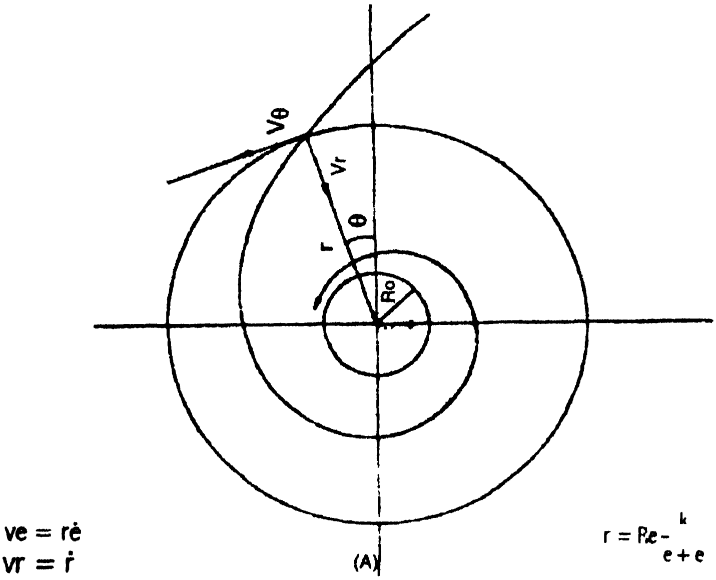

Figure 1. Wind distribution in spiral flow.

Invoking the polar system of co-ordinates such that the origin is at geometrical centre of cyclone and the angle is measured in the positive direction from the arbitrary chosen fixed line OX. Let the air masses affected by the circulation at a radius R_1 have an angular velocity

$$\omega = \frac{d\theta}{dt} \qquad \text{at } r = R_1 \tag{6}$$

The angular momentum say M of unit mass of air associated with this circulation at the point (R_1, θ_1) is

$$M = R_1^2\omega + \frac{fR_1^2}{2} = R_1^2\left[\omega + \frac{f}{2}\right] \tag{7}$$

We take $(\omega + f/2)$ as ω_1 for simplicity to deal in the calculations where f is the coriolis parameter.

It is assumed that the air masses travel along the path of the spiral whose path is governed by

$$r = R_0 e^{\frac{K}{\theta+\theta_0}}; \qquad r \neq 0 \tag{8}$$

where k is a positive constant for cyclonic circulation, θ_0 is a reference angle, R_0 is any

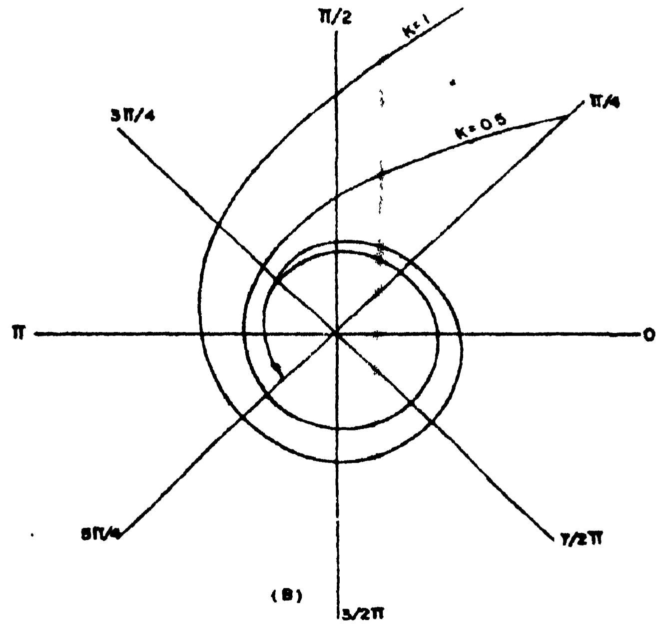

Figure 2. Shape of the spiral for different 'K'

arbitrary constant radius. Figure 1 gives the wind distribution in the given spiral flow. Figure 2 gives the shape of the spiral for different K.
Differentiating (8) with respect to time to time t

$$\frac{dr}{dt} = R_0 e^{\overline{\theta+\theta_0}} \left[\frac{-K}{(\theta+\theta_0)^2}\right] \tag{9}$$

where $\theta = \frac{d\theta}{dt}$

Under the principle of conservation of angular momentum, the angular velocity at any radius r is given by relation

$$\theta = \frac{R_1^2}{r^2}\theta_1 \qquad r \neq 0 \tag{10}$$

where R_1 is the outer most radius and q_1 is the angular velocity at radius R_1 Therefore, applying (10) in equation (9), the radial velocity V_r is given by

$$V_r = \frac{dr}{dt} = -\frac{R_1^2}{Kr}\left[\log\frac{r}{R_0}\right]^2 \theta_1; \qquad r \neq 0. \tag{11}$$

Therefore it can be seen from the equation (11), that V_r tends to zero at radius R_0 i.e., the motion becomes circular at a radius R_0, which can be taken as radius of maximum wind in a cyclonic storm. For $r < R_0$, flow does exist because V_r can have positive direction. Since has to be negative to satisfy the condition of (8) for radius $r < R_0$, the flow inside the radius R_0 becomes outward.

Let V_θ be the azimuthal velocity of the circulation. By invoking equation (10), one has

$$V_\theta = r\theta = \frac{R_1^2}{r}\theta_1; \qquad r \geq R_0$$
$$= r\omega_1 \qquad \text{for} \quad r < R_0 \tag{12}$$

where ω_1 is the angular velocity of the circulation inside the radius of maximum wind R. Differentiating eqn. (11) with respect to radius r

$$\frac{\partial V_r}{\partial r} = \frac{R_1^2}{K_r^2}\theta_1\left[\log\frac{r}{R_0} - 2\right]\log\frac{r}{R_0}; \qquad r \neq 0 \tag{13}$$

$\frac{\partial V_r}{\partial r}$ is negative when $r < R_0 e^2$ which suggests that radian velocity decreases inward from the radius $r = R_0 e^2$ and increases away from the radius.

The equation (13) also be written as

$$\frac{\partial V_r}{\partial r} = -\frac{\partial V_\theta}{K\partial r}\left[\log\frac{r}{R_0} - 2\right]\log\frac{r}{R_0}; \qquad r \neq 0 \tag{14}$$

Therefore, the rate of change of V_θ and V_r are opposite to direction for radius $r > R_0 e^2$ and V_r decreases in the same direction for radius $r > R_0 e^2$.

Under the present assumption, the two dimensional equation in polar co-ordinates is given by

$$V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} = -\alpha\frac{\partial P}{\partial r} + fV_\theta \tag{15}$$

$$V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} - \frac{V_\theta V_r}{r} = -\frac{\alpha}{r}\frac{\partial P}{\partial \theta} - fV_r \qquad r \neq 0 \tag{16}$$

Substituting the value of V_θ, V_r in equation (15) yields

$$\frac{-R_1^4\theta_1}{K^2 R_0^3 x^3}\left[\{\log x - 2\}\{\log x\}^3 + K^2\right] = -\frac{\alpha}{R_0}\frac{\partial P}{\partial x} + fR_1^2\frac{\theta_1}{R_0 x}$$

where

$$x = \frac{r}{R_0}; \qquad r \neq 0 \tag{17}$$

From equation (17) at x=1, the equation becomes

$$-\alpha\frac{\partial P}{\partial r} + fV_\theta = \frac{V_\theta^2}{R_0}; \quad r \neq 0 \tag{18}$$

which is the gradient wind equation. Therefore, under present assumption the follow becomes gradient at the radius of maximum wind. But when V_θ becomes so large such that fV_θ is negligible to $\frac{V_\theta^2}{R_0}$ i.e. centrifugal force term, then the flow becomes cyclostrophic.

Even if we consider that $\frac{\partial V_r}{\partial \theta}$ are negligible the above arguments are valid.

$$V_r\frac{\partial V_\theta}{\partial r}=\frac{R_1^4\theta_1^2}{KR_0^3x^3}\left[\log\frac{r}{R_0}\right]^2=-V_\theta\frac{V_r}{r} \tag{19}$$

$$\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}=-V_\theta\frac{V_r}{r};\qquad r\neq 0. \tag{20}$$

Substituting the values of V_θ V_r and invoking equation (19) and (20) in (16) gives

$$-\frac{V_\theta V_r}{r}=-\frac{\alpha}{r}\frac{\partial P}{\partial \theta}-fV_r \tag{21}$$

$$-\alpha\frac{\partial P}{\partial \theta}=V_r\left[V_\theta-fr\right];\qquad r\neq 0. \tag{22}$$

From equation (19) (20) (21) and (22) it can be inferred that the angular pressure gradient is related to my one of the angular variations of radian and azimuthal velocity only. But at $r=R_0$, the pressure is symmetric with respect to the centre of the system and in respect of other radius, pressure is not symmetric. If either of V_θ or V_r is symmetric with respect to centre, then the pressure will also be symmetric. But in the case of spiral motion V_θ, V_r cannot be symmetric about the centre. Hence asymmetry of pressure is essential for a spiral flow.

Under this assumption, the divergence is given by the relation

$$D=\nabla.V=\frac{\partial V_r}{\partial r}+\frac{V_r}{r}+\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

$$=\frac{\Omega}{x^3}\left[\{\log x\}^2-2\log x\right];\qquad r\neq 0 \tag{23}$$

where $x=\frac{r}{R_0}$ and $\Omega=\frac{R_1^2\theta_1}{R_0^2}$.

To find out the extreme values of D, differentiating with respect to x gives (24)

$$\frac{dD}{dx}=-2\frac{\Omega}{e^{3t}}\left[t^2-3t+1\right]$$

where $t=\log x$.

$$\frac{dD}{dx}=0\Rightarrow t^2+3t+1=0\ \textit{ and hence } t=\left[\frac{3+\sqrt{5}}{2}\right]$$

Therefore, the extreme value of D is reached at $r = R_0 e^{\frac{3\pm\sqrt{5}}{2}}$

Maximum of D occurs at $r = R_0 e^{\frac{3\pm\sqrt{5}}{2}}$ and minimum at $r = R_0 e^{\frac{3-\sqrt{5}}{2}}$

Therefore maximum convergence takes place at $R_0 e^{\frac{3-\sqrt{5}}{2}}$.

It is worthy to mention here, that the contribution of convergence due to azimuthal wind is compensated by the term V_r/r. That is the convergence under the condition solely depends on the radial gradient of radial wind *i.e.*, the amount of air accumulated due to the decrease in radial wind towards the centre is attributed to the vertical acceleration.

$$D = \frac{\partial V_r}{\partial r} \tag{25}$$

The equation of motion along the vertical direction is

$$V_r \frac{\partial W}{\partial r} + \frac{V_\theta}{r}\frac{\partial W}{\partial \theta} + W\frac{\partial W}{\partial z} = -g - \alpha\frac{\partial P}{\partial z} \tag{26}$$

$$\frac{\partial W}{\partial r}\left[V_r + \frac{V_\theta}{r}\frac{\partial r}{\partial \theta}\right] W\frac{\partial W}{\partial Z} = -g - \alpha\frac{\partial P}{\partial z}. \tag{27}$$

The horizontal divergence is equal to the vertical convergence and $\frac{V_\theta}{r}\frac{\partial W}{\partial \theta} = V_r$

therefore

$$2V_r\frac{\partial W}{\partial r} - W\frac{\partial V_r}{\partial r} = -g - \alpha\frac{\partial P}{\partial z} \quad r \neq 0. \tag{28}$$

From equation (27), under the hydrostatic assumption, the advection of radial wind by vertical velocity is equal to twice the advection of vertical velocity by radial wind *i.e.*, in a cyclonic circulation radial velocity is negative and if it is assumed to be positive then they are in opposite directions. Solving (28) in terms of r and hydrostatic assumption, to find out the vertical velocity then,

$$W = \frac{K_1}{\sqrt{r}}\log\frac{r}{R_0}; \qquad r \neq 0 \tag{29}$$

where K_1 is a constant.

4. Results

It can be noticed that W becomes zero at the radius of maximum wind and also at large radius. Inside the RMW it becomes negative *i.e.*, downward motion is explained in the eye of the storm except at the centre of the circulation where the function is not defined. That fact is well observed in the case of cyclonic storms which crossed the east coast of India as seen in Figures 3a, 3b,3c. The flatness of the cumulative rainfall at the time of maximum wind explain the non coincidence of RMW and maximum rainfall region. That is at radius of maximum wind, rainfall ceases. It was observed in case of Tuticorin cyclone of 1992, when, at the time of strong winds the rains ceased. In the case of Chennai cyclone

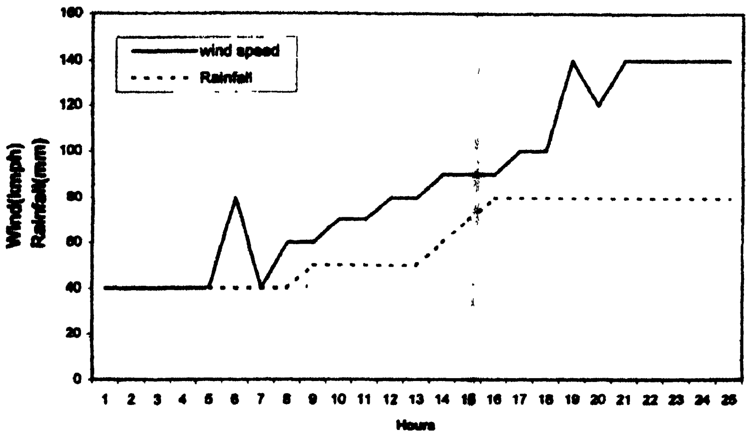

Figure 3a. Wind speed and cumulative rainfall at the time of crossing of May 1990 Storm at Machilipattinam.

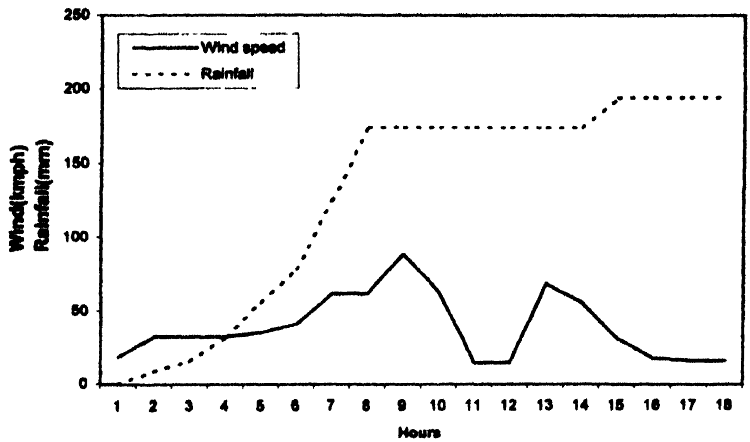

Figure 3b. Wind speed and cumulative rainfall at the time of crossing of December 1993 Storm at Karaikal.

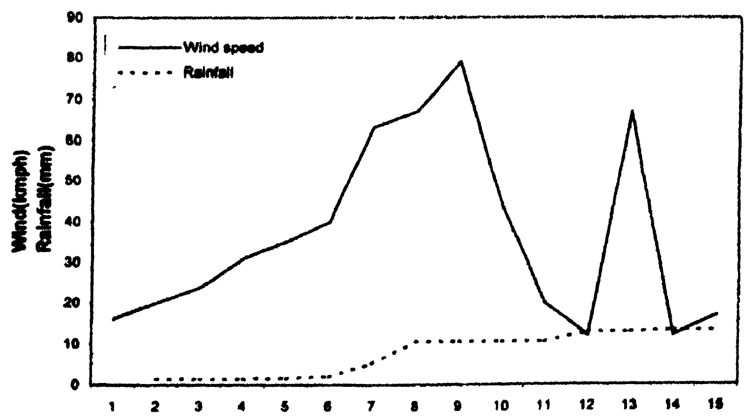

Figure 3c. Wind speed and cumulative rainfall at the time of crossing of December 1993 Storm at Chennai.

of 1994 also, when severe cyclone strength wind started at 0200 UTC of 31 Oct 94, the rainfall stopped over Madras. The rain resumed only at 0700 UTC after the second portion of the RMW crossed the station at 0645 UTC.

In this paper path of approach of wind in a cyclonic storm is taken as spiral as mentioned in equation(8) Under these concept radial wind, tangential wind and wind and pressure relationship is established. Under these circumstances the equation (13) reveals that radian wind initially increases with radius upto a distance of R_e^2 and then decreases towards centre and equal to zero at RMW. Due to this decrease of radial wind near RMW the convergence is very large near the eye of the storm. From equation (19), (20), (21) and (22) it can be inferred that the angular pressure gradient is a function of any one of the angular variations of radial and tangential velocity, but at $r = R_0$, the pressure is symmetric with respect to the centre of the system and the tangential velocity is also symmetric with respect to centre. In respect of other radius, pressure is not symmetric. If either of V_θ or V_r is symmetric with respect to centre then the pressure will also be symmetric, but in the case of spiral motion V_θ or V_r cannot be symmetric about the centre. Hence asymmetry of pressure is essential for a spiral flow.

5. Conclusions

In a circular isobaric field, gradient flow is not possible in an unstable atmosphere Under the assumption of spiral flow in a cyclonic storm many aspects of observed features have been tried to be explained here and the results are as follows:

1. The radian wind becomes zero both at the radius of maximum wind and at a very large radii.
2. Radial wind initially increases with radius upto a distance of R_e^2 and then decreases towards centre and equal to zero at RMW *i.e.*, the wind towards the centre increases upto the radius equal to R_e^2 and then it decreases to zero at $r = R_e$
3. Inside the eye, the structure of the spiral demands the outward radial flow which is an observed fact exactly at the origin where the radial wind is not defined.
4. The pressure gradient $-\alpha(\partial P / \partial r)$ is found to be logarithmic function of r / R_0 and steeply increases in the proximity of the radius of maximum wind and is negligible at $r = R_0$. The flow becomes gradient at the radius of maximum wind. But if v/r>>fv, the flow becomes cyclostrophic due to the negligible value of coriolis force, that is, circular flow is observed only at the radius of maximum wind. The azimuthal component of pressure gradient $-\alpha(\partial P / \partial\theta)$ is nonzero outside the RMW and vanishes at the radius of maximum wind. This gradient is a linear function of radial wind. The symmetry in the pressure distribution is controlled by the variation of radial wind and at RMW radial wind happens to be zero and the pressure distribution is symmetric at this radius.
5. The contribution of divergence by the azimuth wind is compensated by the term $\partial V_r / \partial r$ so that the whole divergence in the system depends on the factor $\partial V_r / \partial r$

only. It is found that the divergence has two extreme values namely one at $R_0 e^{\frac{3-\sqrt{5}}{2}}$ of which the second one has maximum convergence.

6. By the assumption under cylindrical co-ordinate system the vertical component of the equation of motion suggests that the advection of the vertical velocity by the radial wind is compensated by the advection of the radial wind by the vertical velocity under the hydrostatic assumption. The vertical velocity is obtained by the relation $W = \frac{K_1}{\sqrt{r}} \log \frac{r}{R_0}$ where K_1 is a constant. The vertical velocity at RMW is zero and inside the RMW it is negative which is an observed fact in cyclonic storms.

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